3. Motion in a Plane

Can you recall?

1. What is meant by motion?
2. What is rectilinear motion?
3. What is the difference between displacement and distance travelled?
4. What is the difference between uniform and nonuniform motion?

3.1 Introduction:

We see objects moving all around us. Motion is a change in the position of an object with time. We have come across the motion of a toy car when pushed along some particular direction, the motion of a cricket ball hit by a batsman for a sixer and the motion of an aeroplane from one place to another. The motion of objects can be divided in three categories: (1) motion along a straight line, i.e., rectilinear motion, (2) motion in two dimensions, i.e., motion in a plane and, (3) motion in three dimensions, i.e., motion in space. The above cited examples correspond to three types of motions, respectively. You have studied rectilinear motion in earlier standards. In rectilinear motion the force acting on the object and the velocity of the object both are along one and the same line. The distances are measured along the line only and we can indicate distances along the +ve and –ve axes as being positive and negative, respectively. The study of the motion of an object in a plane or in space becomes much easier and the corresponding equations become more elegant if we use vector quantities. In this Chapter we will first recall basic facts about rectilinear motion. We will use vector notation for this study as it will be useful later when we will study the motion in two dimensions. We will then study the motion in two dimensions which will be restricted to projectile motion only. Circular motion, i.e., the motion of an object around a circular path will be introduced here and will be studied in detail in the next standard.

3.2 Rectilinear Motion:

Consider an object moving along a straight line. Let us assume this line to be along the x-axis. Let $\vec{x}_1$ and $\vec{x}_2$ be the position vectors of the body at times t_1 and t_2 during its motion. The following quantities can be defined for the motion.

1. **Displacement:** The displacement of the object between t_1 and t_2 is the difference between the position vectors of the object at the two instances. Thus, the displacement is given by

$$\vec{s} = \Delta\vec{x} = \vec{x}_2 - \vec{x}_1 \quad \text{--- (3.1)}$$

Its direction is along the line of motion of the object. Its dimensions are that of length. For example, if an object has travelled through 1 m from time t_1 to t_2 along the +ve x-direction, the magnitude of its displacement is 1 m and its direction is along the +ve x-axis. On the other hand, if the object travelled along the +ve y direction through the same distance in the same time, the magnitude of its displacement is the same as before, i.e., 1 m but the direction of the displacement is along the +ve y-axis.

2. **Path length**: This is the actual distance travelled by the object during its motion. It is a scalar quantity and its dimensions are also that of length. If an object travels along the x-axis from x = 2 m to x = 5 m then the distance travelled is 3 m. In this case the displacement is also 3 m and its direction is along the +ve x-axis. However, if the object now comes back to x = 4, then the distance through which the object has moved increases to 3 + 1 = 4 m. Its initial position was x = 2 m and the final position is now x = 4 m and thus, its displacement is $\Delta x = 4 - 2 = 2$ m, i.e., the magnitude of the displacement is 2 m and its direction is along the +ve x-axis. If the object now moves to x =1, then the distance travelled, i.e., the path length increases to 4 + 3 =

7 m while the magnitude of displacement becomes 2 – 1 = 1 m and its direction is along the negative *x*-axis.

3. **Average velocity**: This is defined as the displacement of the object during the time interval over which average velocity is being calculated, divided by that time interval. As displacement is a vector quantity, the velocity is also a vector quantity. Its dimensions are $[L^1 M^0 T^{-1}]$.

   If the position vectors of the object are $\vec{x}_1$ and $\vec{x}_2$ at times t_1 and t_2 respectively, then the average velocity is given by

$$\vec{v}_{av} = \frac{\vec{x}_2 - \vec{x}_1}{(t_2 - t_1)} \qquad \text{--- (3.2)}$$

   For example, if the positions of an object are $x = +2$ m and $x = +4$ m at times $t = 0$ and $t = 1$ minute respectively, the magnitude of its average velocity during that time is $v_{av} = (4 - 2)/(1- 0) = 2$ m per minute and its direction will be along the +ve *x*-axis, and we write $\vec{v}_{av} = 2\hat{i}$ m/min where $\hat{i}$ is a unit vector along *x*-axis.

4. **Average speed**: This is defined as the total path length travelled during the time interval over which average speed is being calculated, divided by that time interval.

   Average speed = v_{av} = path length/time interval. It is a scalar quantity and has the same dimensions as that of velocity, i.e., $[L^1 M^0 T^{-1}]$.

   If the rectilinear motion of the object is only in one direction along a line, then the magnitude of its displacement will be equal to the distance travelled and so the magnitude of average velocity will be equal to the average speed. However if the object reverses its direction (the motion remaining along the same line) then the magnitude of displacement will be smaller than the path length and the average speed will be larger than the magnitude of average velocity.

5. **Instantaneous velocity**: Instantaneous velocity of an object is its velocity at a given instant of time. It is defined as the limiting value of the average velocity of the object over a small time interval (Δt) around *t* when the value of the time interval (Δt) goes to zero.

$$\vec{v} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{x}}{\Delta t}\right) = \frac{d\vec{x}}{dt}, \qquad \text{--- (3.3)}$$

   $\frac{d\vec{x}}{dt}$ being the derivative of $\vec{x}$ with respect to *t* (see Chapter 2).

6. **Instantaneous speed:** Instantaneous speed is the speed of an object at a given instant of time *t*. It is the limiting value of the average speed of the object taken over a small time interval (Δt) around *t* when the time interval goes to zero. In such a limit, the path length will be equal to the magnitude of the displacement and so the instantaneous speed will always be equal to the magnitude of the instantaneous velocity of the object.

Always Remember:

For uniform rectilinear motion, i.e., for an object moving with constant velocity along a straight line

1. The average and instantaneous velocities are equal.
2. The average and instantaneous speeds are the same and are equal to the magnitude of the velocity.

For nonuniform rectilinear motion

1. The average and instantaneous velocities are different.
2. The average and instantaneous speeds are different.
3. The average speed will be different from the magnitude of average velocity.

Example 3.1: A person walks from point P to point Q along a straight road in 10 minutes, then turns back and returns to point R which is midway between P and Q after further 4 minutes. If PQ is 1 km, find the average speed and velocity of the person in going from P to R.

Solution: The path length travelled by the person is 1.5 km while the displacement is the distance between R and P which is 0.5 km. The time taken for the motion is 14 min.

The average speed = 1.5 / 14 = 0.107 km/min = 6.42 km/hr.

The magnitude of the average velocity = 0.5/14 = 0.0357 km/min = 2.142 km/hr.

Graphical Study of Motion

We can study the motion of an object by using graphs showing its position as a function of time. Figure 3.1 shows the graphs of position as a function of time for five different types of motion of an object. Figure 3.1(a) shows an object at rest, for which the *x-t* graph is a horizontal straight line. Since the position is not changing, displacement of the object zero. Velocity is displacement (which is zero) divided by time interval or the derivative of displacement with respect to time. It can be obtained from the slope of the line plotted in the figure which is zero.

Figure 3.1(b) shows *x-t* graph for an object moving with constant velocity along the +ve *x*-axis. Since velocity is constant, displacement is proportional to elapsed time. The slope of the straight line is +ve, showing that the velocity is along the +ve *x*-axis. As the motion is uniform, the average velocity is same as the instantaneous velocity at all times. Also, the speed is equal to the magnitude of the velocity.

Figure 3.1(c) shows the *x-t* graph for a body moving with uniform velocity but along the -ve *x*-axis, the slope of the line being -ve. Figure 3.1(d) shows the *x-t* graph of an object having oscillatory motion with constant speed. The direction of velocity changes from +ve to -ve and vice versa over fixed intervals of time.

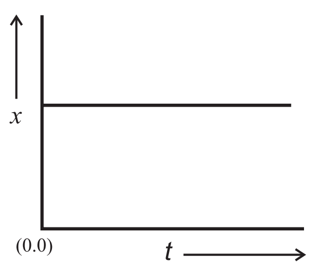


Fig 3.1 (a): Object at rest.

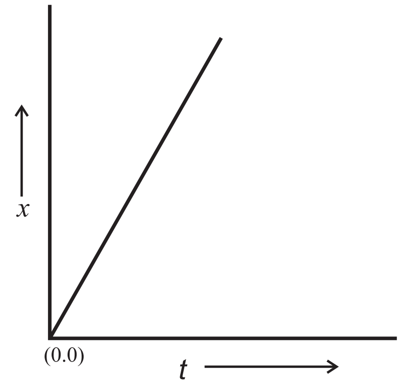


Fig 3.1 (b): Object with uniform velocity along +ve *x*-axis.

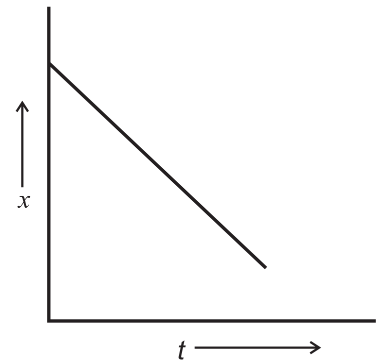


Fig 3.1 (c): Object with uniform velocity along -ve *x*-axis.

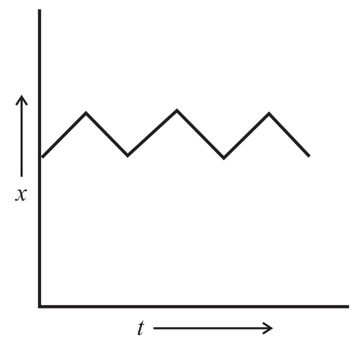


Fig 3.1 (d): Object performing oscillatory motion.

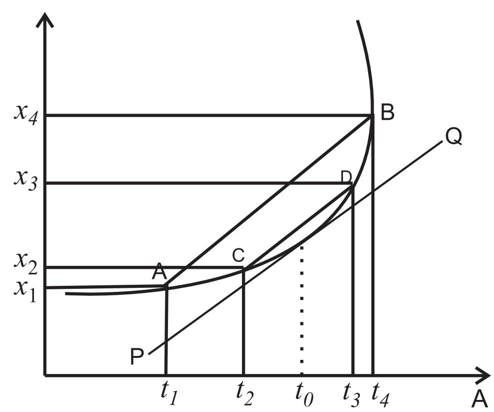


Fig.3.1 (e): Object in nonuniform motion.

Figure 3.1(e) shows the motion of an

object with nonuniform velocity. Its velocity changes with time and, therefore, the average and instantaneous velocities are different. Figure shows the average velocity over time interval from t_1 to t_4 around time t_0, which can be seen from Eq. (3.2) to be the slope of line AB. For a smaller time interval from t_2 to t_3, the average velocity is the slope of the line CD. If we keep reducing the time interval around t_0, we will ultimately come to a limit, when the time interval will go to zero and lines AB, CD... will go over to the tangent to the curve at t_0. The instantaneous velocity at t_0 will thus be equal to the slope of the tangent PQ at t_0 (see Eq. (3.3)).

7. Acceleration: Acceleration is defined as the rate of change of velocity with time. It is a vector quantity and its dimensions are $[L^1 M^0 T^{-2}]$. The average acceleration of an object having velocities $\vec{v}_1$ and $\vec{v}_2$ at times t_1 and t_2 is given by

$$\vec{a} = \frac{(\vec{v}_2 - \vec{v}_1)}{(t_2 - t_1)} \quad \text{--- (3.4)}$$

Instantaneous acceleration is the limiting value of the average acceleration when the time interval goes to zero. It is given by

$$\vec{a} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{v}}{\Delta t}\right) = \frac{d\vec{v}}{dt} \quad \text{--- (3.5)}$$

The instantaneous acceleration at a given time is the slope of the tangent to the velocity versus time curve at that time. Figure 3.2 shows the velocity versus time (v - t) graphs for four different cases. Figure 3.2(a) represents the motion of an object with zero acceleration, i.e., constant velocity. The shaded area under the velocity-time graph over some time interval t_1 to t_2, shown in Figs. 3.2(a) is equal to $v_0 (t_2 - t_1)$ which is the magnitude of the displacement of the object from t_1 to t_2. Figure 3.2(b) is the velocity-time graph for an object moving with constant +ve acceleration (magnitude of velocity uniformly increasing with time). Figure 3.2(c) shows similar motion but the object has -ve acceleration, i.e., the acceleration is opposite to the direction of velocity which, therefore, decreases uniformly with time. The area under both the curves between two instants of time is the displacement of the object during that time interval (as shown below). Figure 3.2(d) shows the motion of an object having nonuniform acceleration. The average acceleration between t_1 and t_2 around t_0 and the instantaneous accelerations at t_0 for the object are shown by straight lines AB and CD respectively.

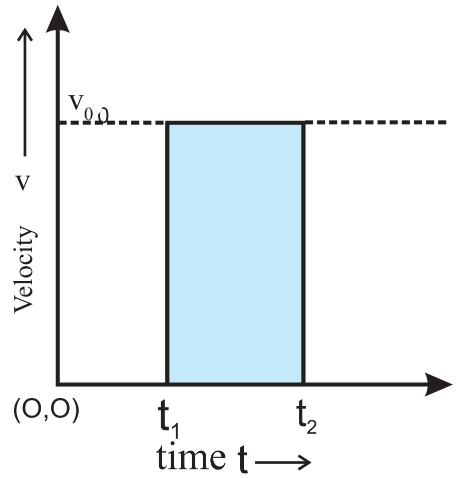


Fig 3.2 (a): Object moving with constant velocity.

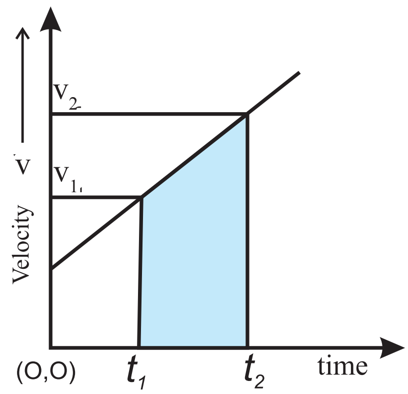


Fig 3.2 (b): Object moving with velocity (v) along +ve x-axis with uniform acceleration along the same direction.

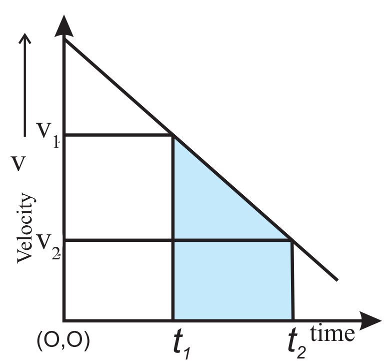


Fig 3.2 (c): Object moving with velocity (v) with negative uniform acceleration.

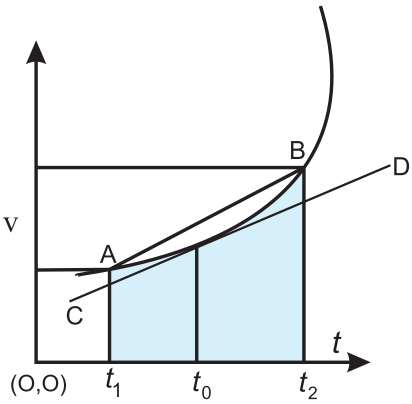


Fig. 3.2 (d): Object moving with nonuniform acceleration.

The area under the velocity-time curves in Figs. 3.2(a) to (d) can be written using the definition of integral given in Chapter 2 as

$$\text{Area} = \int_{t_1}^{t_2} \text{v}\, dt = \int_{t_1}^{t_2} \frac{dx}{dt} dt = \int_{t_1}^{t_2} dx = x(t_2) - x(t_1) \quad \text{--- (3.6)}$$

= displacement of the object from t_1 to t_2.

Always Remember:

For uniform acceleration, for a rectilinear motion:

1. Velocity-time graph is linear.
2. The area under the velocity-time graph between two instants of time t_1 and t_2 gives the displacement of the object during that time interval.
3. The slope of the velocity-time graph is the acceleration of the object

For nonuniform acceleration in a rectilinear motion:

1. Velocity-time graph is nonlinear.
2. The area under the velocity-time graph between two instants of time t_1 and t_2 gives the displacement of the object during that time interval.
3. The instantaneous acceleration of the object at a given time is equal to the slope of the tangent to the curve at that point.

While using the concept of area under the curve, the origin of the velocity axis (for v-*t* graph) must be zero.

Equations of Motion for Uniform Acceleration:

We can graphically derive Newton's equations of motion for an object moving with uniform acceleration. Consider an object having position $x = 0$ at $t = 0$. Let the velocity at $t = 0$ be u and at time t be v. The graphical representation of motion is shown in Fig. 3.3. The acceleration is given by the slope of the line AB. Thus,

$$\text{Acceleration, } a = \frac{\text{v} - u}{t - 0} = \frac{\text{v} - u}{t}$$

$$\therefore \text{v} = u + at \quad \text{--- (3.7)}$$

This is the first equation of motion.

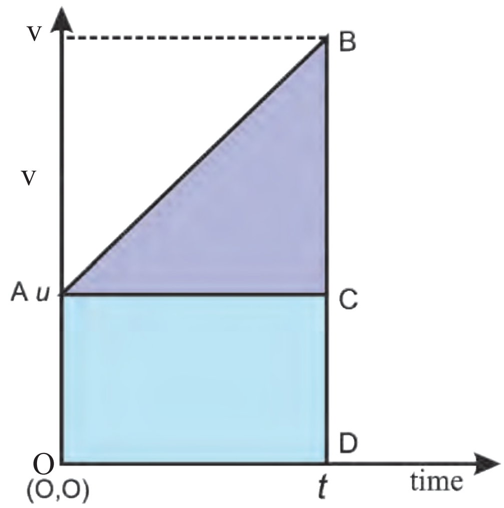


Fig.3.3: Derivation of equation of motion for motion with uniform acceleration.

As we know, the area under the curve in velocity-time graph is the displacement of the object. Thus displacement s = area of the quadrilateral OABD. = area of triangle ABC + area of rectangle OACD.

$$= \frac{1}{2}(v - u)t + ut$$

Using Eq. (3.7), $s = ut + \frac{1}{2}at^2$ --- (3.8)

This is the second equation of motion.

As the acceleration is constant, the velocity is increasing linearly with time and we can use average velocity v_{av}, to calculate the displacement using Eq. (3.7) as

$$s = \text{v}_{av} t = \left(\frac{\text{v} + u}{2}\right) t = \frac{(\text{v} + u)(\text{v} - u)}{2a}$$

$$\therefore s = (\text{v}^2 - u^2)/(2a)$$

$$\therefore \text{v}^2 - u^2 = 2a.s \quad \text{--- (3.9)}$$

This is the third equation of motion. Vector notation was not included here as the motion was rectilinear.

The most common example of uniform rectilinear motion with uniform acceleration of an object in day to day life is a freely falling body. When a body starts with zero velocity at a certain height from the ground and falls under the influence of the gravity of the Earth , it is said to be in free fall. The only other force that acts on it is that of the air resistance or friction. For displacements of a few metres, this force is too small and can be neglected. The acceleration of the body is the acceleration due to gravity which is along the vertical direction and can be assumed to be constant over distances which are small compared to the radius of the Earth . Thus the velocity and acceleration are both along the vertical direction and the motion is a uniform rectilinear motion with uniform acceleration.

Do you know ?

The distance travelled by an object starting from rest and having a uniform acceleration in successive seconds are in the ratio 1:3:5:7... Consider a freely falling object. Let us calculate the distances travelled by it in equal intervals of time t_0 (say). This can be done using the second equation of motion $s = u\, t_0 + (1/2)\, g\, t_0^2$. The initial velocity is zero. Therefore, the distance travelled in the first t_0 interval = $(1/2)\, g\, t_0^2$. For simplification let us write $(1/2)\, g$ = A. Then the distance travelled in the first t_0 time interval = $d_1 = At_0^2$. In the time interval $2t_0$, the distance travelled = $A(2t_0)^2$. Hence, the distance travelled in the second t_0 interval is $d_2 = A(4t_0^2 - t_0^2) = 3A\, t_0^2 = 3\, d_1$. The distance travelled in time interval $3t_0 = A(3t_0)^2$. Thus, the distance travelled in the 3rd t_0 interval = $d_3 = A(9t_0^2 - 4t_0^2) = 5A\, t_0^2 = 5d_1$. Continuing, one can see that the distances d_1, d_2, d_3 .. are in the ratio 1:3:5:7... This is true for any rectilinear motion, starting from rest, with positive uniform acceleration.

Example 3.2: A stone is thrown vertically upwards from the ground with a velocity 15 m/s. At the same instant a ball is dropped from a point directly above the stone from a height of 30 m. At what height from the ground will the stone and the ball meet and after how much time? (Use $g = 10$ m/s^2 for ease of calculation).

Solution: Let us assume that the stone and the ball meet after time t_0. The distances (not displacements) travelled by the stone and the ball in that time can be obtained from Eq. (3.8) as

$$s_{stone} = 15\, t_0 - \frac{1}{2}\, g\, t_0^2$$

$$s_{ball} = \frac{1}{2}\, g\, t_0^2$$

When they meet, $s_{stone} + s_{ball} = 30$

$$15\, t_0 - \frac{1}{2}\, g\, t_0^2 + \frac{1}{2}\, g\, t_0^2 = 30$$

$$t_0 = 30/15 = 2 \text{ s}$$

$$\therefore\ s_{stone} = 15\,(2) - \frac{1}{2}\,(10)\,(2)^2 = 30 - 20 = 10 \text{ m}$$

Thus the stone and the ball meet at a height of 10 m.

8. Relative Velocity: You must have often experienced relative motion. The most striking example is when you are going in a train and another train travelling in the same direction along parallel tracks, overtakes you. If you look at that train, it actually seems to be moving much slower than what your train seemed to move and yet it is overtaking you. On the other hand if your train overtakes another train, travelling on a parallel track in the same direction, and you look at that train, you feel that your train has suddenly slowed down. Why does this happen? This is because when you look at the neighbouring train, you are actually experiencing relative motion, i.e., your motion with respect to the other train or the motion of the other train with respect to you. Thus, in the first case as the other train overtakes you what you perceive is the velocity of the train with respect to you, i.e., the difference in the velocities of the two trains which most often is much smaller than the velocity of your train. In the second case, you are moving faster but when you look at that train you only feel your velocity relative to it and, therefore, your velocity appears to be lower than its actual value. We can define relative velocity of object A with respect to object B as the difference between their velocities, i.e.,

$$v_{AB} = v_A - v_B \qquad \text{--- (3.10)}$$

Similarly, the velocity of B with respect to A is given by

$$v_{BA} = v_B - v_A \qquad \text{--- (3.11)}$$

We assume that at time $t = 0$, A and B were at the same point $x = 0$. As they are travelling with different velocities, the distance between them will go on increasing with time in direct proportion to the difference in their velocities, i.e., the relative velocity between them.

Example 3.3: An aeroplane A, is travelling in a straight line with a velocity of 300 km/hr with respect to Earth. Another aeroplane B, is travelling in the opposite direction with a velocity of 350 km/hr with respect to Earth. What is the relative velocity of A with respect to B? What should be the velocity of a third aeroplane C moving parallel to A, relative to the Earth if it has a relative velocity of 100 km/hr with respect to A?

Solution: Let v_A, v_B and v_C be the velocities of the three planes relative to the Earth. Relative velocity of A with respect to B = $v_{AB} = v_A - v_B$ = 300 – (-350) = 650 km/hr

Relative velocity of C with respect to A = v_{CA} = $v_C - v_A$ = 100 km/hr.

Thus, $v_C = v_{CA} + v_A$ = 400 km/hr

3.3 Motion in Two Dimensions-Motion in a Plane:

So far we were considering rectilinear motion of an object. The direction of motion of the object was always along one straight line. Now we will consider the motion of an object in two dimensions, i.e., along a plane. Here, the direction of the force acting on an object will not be in the same line as its initial velocity. Thus, the velocity and acceleration will have different directions. For this reason we have to use vector equations. The definitions of various terms given in section 3.2 will remain valid except that the magnitude of the average velocity and the value of average speed will be different as the magnitude of the displacement need not be equal to the path length. For example, if a particle travels along a circle and comes back to its original position, its displacement will be zero but the path length will be equal to the circumference of the circle.

3.3.1 Average and Instantaneous Velocities:

For studying the motion of an object in two dimensions, for simplicity, we will take the plane to be the *x-y* plane. To describe the position of an object in this plane we will have to specify, both its *x* and *y* coordinates. The definitions of displacement, average and instantaneous velocities, average and instantaneous speeds and acceleration will be the same as those for rectilinear motion except that each of these quantities will now have components along the *x* and *y* directions. Let us assume the object to be at point P at time t_1 as shown in Fig. 3.4 (a).

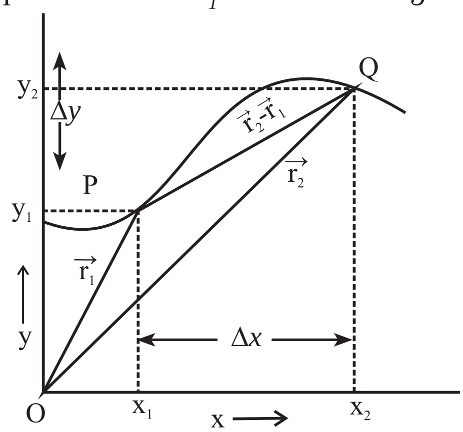


Fig. 3.4 (a) Motion in two dimensions

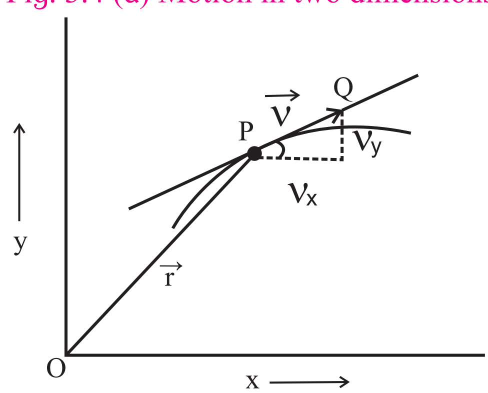


Fig. 3.4 (b) Instantaneous velocity

The position of the object will be described by its position vector $\vec{r}_1$. This can be written in terms of its components along the *x* and *y* axes as

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} \quad \text{--- (3.12)}$$

At time t_2, let the position of the object be Q and its position vector be $\vec{r}_2$

$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} \quad \text{--- (3.13)}$$

The displacement of the particle from t_1 to t_2 shown by PQ, i.e., in time $t = t_2 - t_1$ is given by

$$\overrightarrow{\Delta r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} \quad \text{--- (3.14)}$$

We can write the average velocity of the object as

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \left(\frac{x_2 - x_1}{t_2 - t_1}\right)\hat{i} + \left(\frac{y_2 - y_1}{t_2 - t_1}\right)\hat{j}$$

$$\vec{v}_{av} = (v_{av})_x\hat{i} + (v_{av})_y\hat{j} \quad \text{--- (3.15)}$$

where, $(v_{av})_x = (x_2 - x_1)/(t_2 - t_1)$ and

$$(v_{av})_y = (y_2 - y_1)/(t_2 - t_1) \quad \text{--- (3.16)}$$

Average velocity is a vector whose direction is along $\Delta\vec{r}$ (see Eq. (3.2)), i.e., along the direction of displacement. In terms of its components, the magnitude (v) and direction (the angle θ that the velocity vector makes with the x-axis) can be written as (see Chapter 2)

$$v_{av} = \sqrt{(v_{av})_x^2 + (v_{av})_y^2} \quad \text{and}$$

$$\tan\theta = (v_{av})_y/(v_{av})_x \quad \text{--- (3.17)}$$

Figure 3.4(b) shows the trajectory of an object moving in two dimensions. The instantaneous velocity of the object at point P along the trajectory is along the tangent to the curve at P. This is shown by the vector PQ. Its x and y components v_x and v_y are also shown in the figure.

The instantaneous velocity of the object can be written in terms of derivative as (see Eq. 3.3)

$$\vec{v} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{r}}{\Delta t}\right) = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j} \quad \text{-- (3.18)}$$

The magnitude and direction of the instantaneous velocity are given by

$$v = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}, \quad \text{--- (3.19)}$$

$$\tan\theta = (dy/dt)/(dx/dt) = dy/dx \quad \text{--- (3.20)}$$

which is the slope of the tangent to the curve at the point at which we are calculating the instantaneous velocity.

3.3.2 Average and Instantaneous Acceleration:

Again, the definitions are the same as those for rectilinear motion. Thus, the average acceleration ($\vec{a}_{av}$) of a particle between times t_1 and t_2 can be written as

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \left(\frac{v_{2x} - v_{1x}}{t_2 - t_1}\right)\hat{i} + \left(\frac{v_{2y} - v_{1y}}{t_2 - t_1}\right)\hat{j} \quad \text{- (3.21)}$$

where $\vec{v}_2$ and $\vec{v}_1$ are the velocities of the particle at times t_2 and t_1 respectively.

$$\vec{a}_{av} = (a_{av})_x\hat{i} + (a_{av})_y\hat{j} \quad \text{--- (3.22),}$$

$(a_{av})_x$ and $(a_{av})_y$ being the x and y components of the average acceleration.

The magnitude and direction of the acceleration are given by

$$a_{av} = \sqrt{(a_{av})_x^2 + (a_{av})_y^2} \quad \text{--- (3.23)}$$

and

$$\tan\theta = (a_{av})_y/(a_{av})_x \quad \text{--- (3.24)}$$

The instantaneous acceleration is given by (see Eq. (3.5))

$$\vec{a} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{v}}{\Delta t}\right) = \frac{d\vec{v}}{dt} = \left(\frac{dv_x}{dt}\right)\hat{i} + \left(\frac{dv_y}{dt}\right)\hat{j} \quad \text{-(3.25)}$$

$$= \frac{d}{dt}\left(\frac{dx}{dt}\right)\hat{i} + \frac{d}{dt}\left(\frac{dy}{dt}\right)\hat{j} = \left(\frac{d^2x}{dt^2}\right)\hat{i} + \left(\frac{d^2y}{dt^2}\right)\hat{j} \quad \text{--- (3.26)}$$

Thus, the x and y components of the instantaneous acceleration are respectively given by

$$a_x = d^2x/dt^2 \text{ and } a_y = d^2y/dt^2 \quad \text{--- (3.27)}$$

The magnitude and direction of the instantaneous acceleration are given by

$$a = \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2} \quad \text{--- (3.28),}$$

and

$$\tan\theta = (dv_y/dt)/(dv_x/dt) = dv_y/dv_x \quad \text{--- (3.29)}$$

which is the slope of the tangent to the curve in velocity graph, i.e., a plot of v_y versus v_x.

Example 3.4: The position vectors of three particles are given by

$\vec{x}_1 = (5\hat{i} + 5\hat{j})$ m, $\vec{x}_2 = (5t\hat{i} + 5t\hat{j})$ m and $\vec{x}_3 = (5t\hat{i} + 10t^2\hat{j})$ m as a function of time t. Determine the velocity and acceleration for each, in SI units.

Solution: $\vec{v}_1 = d\vec{x}_1/dt = 0$ as $\vec{x}_1$ does not depend on time t.

Thus, the particle is at rest.

$\vec{v}_2 = d\vec{x}_2/dt = 5\hat{i} + 5\hat{j}$ m/s. $\vec{v}_2$ does not change with time. $\therefore \vec{a}_2 = 0$

$v_2 = \sqrt{5^2+5^2} = 5\sqrt{2}$ m/s, tan θ = 5/5 = 1 or θ = 45°. Thus, the direction of v_2 makes an angle of 45° to the horizontal.

$\vec{v}_3 = d\vec{x}_3/dt = 5\hat{i} + 20t\hat{j}$.

$\therefore v_3 = \sqrt{5^2+(20t)^2}$ m/s. Its direction is along $\theta = \tan^{-1}\left(\frac{20t}{5}\right)$ with the horizontal.

$\vec{a}_3 = \frac{d\vec{v}_3}{dt} = 20\hat{j}$ m/s^2

Thus, the particle 3 is getting accelerated along the *y*-axis at 20 m/s^2.

3.3.3 Equations of Motion for an Object travellinging a Plane with Uniform Acceleration:

We have derived equations of motion for an object in rectilinear motion in section 3.2. We will now derive similar equations for a particle moving with uniform acceleration in two dimensions. Let the initial velocity of the object be $\vec{u}$ at *t* = 0 and its velocity at time *t* be $\vec{v}$. As the acceleration is constant, the average acceleration and the instantaneous acceleration will be equal. By using the definition of acceleration (Eq. (3.21)), we get

$$\vec{a} = (\vec{v} - \vec{u})/(t - 0)$$

$$\text{or } \vec{v} = \vec{u} + \vec{a}t \quad \text{--- (3.30)}$$

which is the same as Eq. (3.7) but is in vector form.

Let the displacement from time *t* = 0 to *t* be $\vec{s}$. This can be calculated from the average velocity of the object during this time. For constant acceleration, $\vec{v}_{av} = \frac{\vec{u}+\vec{v}}{2}$

$$\therefore \vec{s} = (\vec{v}_{av})t = \left(\frac{\vec{u}+\vec{v}}{2}\right)t = \left(\frac{\vec{u}+\vec{u}+\vec{a}t}{2}\right)t$$

$$\therefore \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \quad \text{--- (3.31),}$$

which is the vector form of Eq. (3.8).

Eq. (3.30) and (3.31) can be resolved into their x and y components so as to get corresponding scalar equations as follows.

$$v_x = u_x + a_x t \quad \text{--- (3.32)}$$

and $$v_y = u_y + a_y t \quad \text{--- (3.33)}$$

$$s_x = u_x t + \frac{1}{2}a_x t^2 \quad \text{--- (3.34)}$$

and $$s_y = u_y t + \frac{1}{2}a_y t^2 \quad \text{--- (3.35)}$$

We can see that Eqs. (3.32) and (3.34) involve only the x components of displacement, velocity and acceleration while Eqs. (3.33) and (3.35) involve only the y components of these quantities. Thus the two sets of equations are independent of each other and can be solved independently. We can thus see that the motion along the x direction of an object is completely controlled by the x components of velocity and acceleration while that along the y direction is completely controlled by the y components of these quantities. This makes it easy to study the motion in two dimensions which gets converted to two independent rectilinear motions along two perpendicular directions.

Always Remember:

Motion in two dimensions can be resolved into two independent motions in mutually perpendicular directions.

Example 3.5: The initial velocity of an object is $\vec{u} = 5\hat{i} + 10\hat{j}$ m/s. Its constant acceleration is $\vec{a} = 2\hat{i} + 3\hat{j}$ m/s^2. Determine the velocity and the displacement after 5 s.

Solution:

$\vec{v} = \vec{u} + \vec{a}t$

$= (5\hat{i} + 10\hat{j}) + (2\hat{i} + 3\hat{j})(5) = 15\hat{i} + 25\hat{j}$

$\therefore v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{15^2 + 25^2} = \sqrt{225+625} = \sqrt{850}$

$= 29.15$ m/s

Direction of $\vec{v}$ with *x*-axis is $\tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}$

$\left(\frac{25}{15}\right) = \tan^{-1}(1.667) = 59^o$

$$\vec{s} = \vec{u}\,t + \frac{1}{2}\vec{a}\,t^2$$

$$= \left(5\hat{i} + 10\hat{j}\right)(5) + \frac{1}{2}\left(2\hat{i} + 3\hat{j}\right)5^2$$

$$= 50\hat{i} + (87.5)\hat{j}$$

$$\therefore s = \sqrt{s_x^2 + s_y^2} = \sqrt{50^2 + 87.5^2}$$

$$= \sqrt{2500 + 7656.25}$$

$$= \sqrt{10156.25} = 100.78\text{ m}$$

at $\tan^{-1}\frac{87.5}{50} = 60°15'$ with x-axis.

3.3.4 Relative Velocity:

Relative velocity between two objects moving in a plane can be defined in a way similar to that for objects moving along a straight line. The relative velocity of object A having velocity $\vec{v}_A$, with respect to the object B having velocity $\vec{v}_B$, is given by

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B \qquad \text{--- (3.36)}$$

Similarly, the relative velocity of object B with respect to object A , is given by

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A \qquad \text{--- (3.37)}$$

We can see that the magnitudes of the two relative velocities (v_{AB} and v_{BA}) are equal and their directions are opposite.

Consider a number of objects A, B, C, D ---- Y, Z, moving with respect to the other. Using the symbol v_{AB} for representing the velocity of A relative to B etc, the velocity of A relative to Z can be written as

$$\vec{v}_{AZ} = \vec{v}_{AB} + \vec{v}_{BC} + \vec{v}_{CD} + \ldots + \vec{v}_{XY} + \vec{v}_{YZ}$$

Note the order of subscripts (A→B→C→D---→Z).

Example 3.6: An aeroplane is travelling northward with a speed of 300 km/hr with respect to the Earth, when wind is blowing from east to west at a speed of 100 km/hr. What is the velocity of the aeroplane with respect to the wind?

Solution: Let the velocity of the aeroplane with respect to Earth be $\vec{v}_{AE}$, velocity of wind with respect to Earth be $\vec{v}_{WE}$. The velocity of aeroplane with respect to wind, $\vec{v}_{AW}$ can be determined by the following expression:

$\vec{v}_{AW} = \vec{v}_{AE} + \vec{v}_{EW} = \vec{v}_{AE} - \vec{v}_{WE} = -100\,\hat{i} + 300\,\hat{j}$, considering north along +y axis.

Magnitude of $\vec{v}_{AW} = \sqrt{(10000 + 90000)}$

$= 100\sqrt{10}$ km/hr, and its direction,

$\theta = \tan^{-1}\left(\frac{300}{-100}\right) = 71.6°$ is towards north of east.

3.3.5 Projectile Motion:

Any object in flight after being thrown with some velocity is called a projectile and its motion is called projectile motion. We often see projectile motion in our day-to-day life. Children throw stones towards trees for getting tamarind pods or mangoes. A bowler bowls a ball towards a batsman in cricket, a basket ball player throws a ball towards the basket, all these are illustrations of projectile motion. In this motion, we have objects (projectiles) with given initial velocity, moving under the influence of the Earth's gravitational field. The projectile has two components of velocity, one in the horizontal, i.e., along x-direction and the other in the vertical, i.e., along the y direction. The acceleration due to gravity acts only along the vertically downward direction. The horizontal component of velocity, therefore, remains unchanged as no force is acting in the horizontal direction, while the vertical component changes in accordance with laws of motion with $\vec{a}_x$ being 0 and $\vec{a}_y$ $(= -\vec{g})$ being the downward acceleration due to gravity (upward is positive). Unless stated otherwise, retarding forces like air resistance, etc., are neglected for the projectile motion.

Let us assume that the initial velocity of the projectile is $\vec{u}$ and its direction makes an angle θ with the horizontal as shown in Fig. 3.5. The projectile is thrown from the ground. We take the x-axis along the ground and y-axis in the vertical direction. The horizontal and vertical components of initial velocity are u

cosθ and u sinθ respectively. The horizontal component remains unchanged in absence of any force acting in that direction, while the vertical component changes according to (Eq. 3.33) with $a_y = -g$ and $u_y = u \sin\theta$.

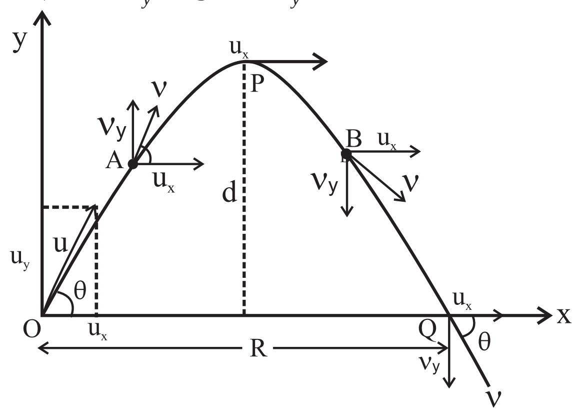


Fig.3.5: Trajectory of a projectile.

Thus, the components of velocity at time t are given by

$$v_x = u_x = u \cos\theta \quad \text{--- (3.38)}$$

$$v_y = u_y - gt = u \sin\theta - gt \quad \text{--- (3.39)}$$

As 0 < θ < 90°, the vertical component initially is in the upward direction. Similarly, the displacements of the projectile in the horizontal and vertical directions at time t, according to Eqs. (3.34) and (3.35) are given by

$$s_x = u\cos\theta.t \quad \text{---- (3.40)}$$

$$s_y = u\sin\theta.t - \frac{1}{2}gt^2 \quad \text{--- (3.41)}$$

The direction of motion of the projectile at any time t makes an angle α with the horizontal which is given by

$$\tan\alpha = v_y(t)/v_x(t) \quad \text{--- (3.42)}$$

The vertical velocity keeps on decreasing as the projectile goes up and becomes zero at certain time. At that time the height of the projectile is maximum. The velocity then starts increasing in the downward direction as the particle is now falling under the Earth 's gravitational field with a constant horizontal component of velocity. After a while the projectile reaches the ground. The trajectory of the object is shown in Fig. 3.5. The projectile is assumed to start from the origin of the coordinate system, O. The point of maximum height is indicated by P and the point where it falls down to the ground is indicated by Q. The horizontal and vertical components of velocity are shown at these points as well as at two intermediate points A and B, on the trajectory of the projectile. Note that the horizontal component of velocity remains the same, i.e., u_x, while the vertical component decreases and becomes zero at P. After that it changes its direction, its magnitude increases and becomes equal to u_y again at Q. The horizontal distance covered by the projectile before it falls to the ground is OQ. We can derive the equation of the trajectory of the projectile as follows.

Let the time taken by the projectile to reach the maximum height be t_0. The trajectory of the object being symmetrical, it can be shown by using equations of motion, that the object will take the same time in going up in air and coming down to the ground. At the highest point P, $t = t_0$ and $v_y = 0$. Using Eq. (3.39),

we get, $0 = u \sin\theta - gt_0$

$$t_0 = (u \sin\theta)/g \quad \text{--- (3.43)}$$

∴ Total time in air = $T = 2t_0$ is the ***time of flight***.

The total horizontal distance travelled by the particle in this time T can be obtained by using Eq. (3.40) as

$$R = u_x . T = u \cos\theta . 2t_0 = u \cos\theta . (2u \sin\theta)/g$$

$$= 2\, u_x u_y/g = u^2(2 \sin\theta \cos\theta)/g$$

$$= u^2 \sin 2\theta/g \quad \text{--- (3.44)}$$

This maximum horizontal distance travelled by the projectile is called the ***horizontal range*** R of the projectile and depends on the magnitude and direction of initial velocity of the projectile as well as the value of acceleration due to gravity at that place.

For maximum horizontal range,

$\sin 2\theta = 1 \;\therefore 2\theta = 90^0 \; or \; \theta = 45^0$

Hence, $R = R_{max} = \frac{u^2}{g}$ for $\theta = 45^0$

The ***maximum height*** H reached by the projectile, having certain value of θ, is the distance travelled along the vertical (y) direction in time t_0. This can be calculated by using Eq. (3.41) as

$$H = u \sin\theta . t_0 - \frac{1}{2} g t_0^2$$

$$= u \sin\theta \left(\frac{u \sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{u \sin\theta}{g}\right)^2$$

$$= \frac{u^2 \sin^2\theta}{2g} = \frac{u_y^2}{2g} \quad \text{--- (3.45)}$$

Do you know ?

All the above expressions of T, R, R_{max} and H are valid if the entire motion is governed only by gravitational acceleration g, i.e., retarding forces like air resistance are absent. However, in reality, it is never so. As a result, time of ascent t_a and time of decent t_d are not equal but $t_a > t_d$. Also, in order to achieve maximum horizontal range for given initial velocity, the angle of projection should be greater than 45^0 and the range is much less than $\frac{u^2}{g}$.

Example 3.7: A stone is thrown with an initial velocity components of 20 m/s along the vertical, and 15 m/s along the horizontal direction. Determine the position and velocity of the stone after 3 s. Determine the maximum height that it will reach and the total distance travelled along the horizontal on reaching the ground. (Assume $g = 10$ m/s^2)

Solution: The initial velocity of the stone in x-direction = $u\cos\theta$ = 15 m/s and in y-direction = $u \sin\theta$ = 20 m/s.

After 3 s, $v_x = u\cos\theta$ = 15 m/s and $v_y = u \sin\theta - gt$ = 20 – 10(3)= - 10 m/s = 10 m/s downwards.

$$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 10^2}$$
$$= \sqrt{225 + 100} = \sqrt{325}$$
$$= 18.03 \text{ m/s}$$

$\tan\alpha = v_y / v_x = 10/15 = 2/3$

$\therefore \alpha = \tan^{-1}(2/3) = 33° \, 41'$ with the horizontal.

$s_x = (u\cos\theta)\, t = 15\times3 = 45$ m,

$s_y = (u\sin\theta)\, t - \frac{1}{2} gt^2 = 20 \times 3 - 5(3)^2 = 15$ m.

Thus the stone will be at a distance 45 m along horizontal and 15 m along vertical direction from the initial position after time 3 s. The velocity is 18.03 m/s making an angle 33° 41' with the horizontal.

The maximum vertical distance travelled is given by $H = (u \sin\theta)^2/(2g) = 20^2/(2 \times 10) = 20$ m

Maximum horizontal distance travelled

$R = 2.u_x.u_y/g = 2(15)(20)/10 = 60$ m

Equation of motion for a projectile

We can derive the equation of motion of the projectile which is the relation between the displacements of the projectile along the vertical and horizontal directions. This can be obtained by eliminating t between the equations giving these displacements, i.e., Eqs. (3.40) and (3.41).

As the projectile starts from $\vec{x} = 0$, we can write $s_x = x$ and $s_y = y$.

$$\therefore s_x = (u\cos\theta)t \quad \therefore t = \frac{s_x}{u\cos\theta} = \frac{x}{u\cos\theta}$$

$$\therefore y = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$= (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\therefore y = (\tan\theta)x - \frac{1}{2}\left(\frac{g}{u^2\cos^2\theta}\right)x^2 \quad \text{--- (3.46)}$$

This is the equation of the trajectory of the projectile. Here, u and θ are constants for the given projectile motion. The above equation is of the form

$$y = Ax + Bx^2 \quad \text{--- (3.47)}$$

which is the equation of a parabola. Thus, the path, i.e., the trajectory of a projectile is a parabola.

3.4 Uniform Circular Motion:

An object moving with *constant speed* along a circular path is said to be in *uniform* circular motion (UCM). Such a motion is only possible if its velocity is *always tangential* to its circular path, *without change in its magnitude*.

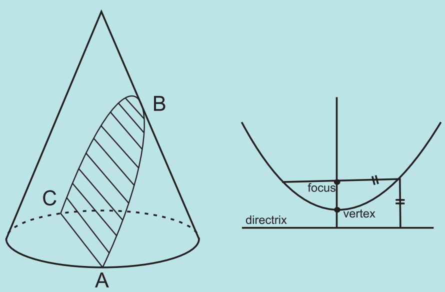
To change the direction of velocity, acceleration is a must. However, if the acceleration or its component is in line with the velocity (along or opposite to the velocity), it will *always* change the speed (magnitude of velocity) in which case it will not continue its uniform circular motion. In order to achieve both these requirements, the acceleration must be (i) perpendicular to the tangential velocity, (ii) of constant magnitude and (iii) always directed towards the centre of the circular trajectory. Such an acceleration is called *centripetal* (centre seeking) acceleration and the force causing this acceleration is *centripetal* force.

Thus, in order to realize a circular motion, there are two requirements; (i) *tangential velocity* and (ii) *centripetal force* of *suitable constant magnitude*.

An example is the motion of the moon going around the Earth in an early circular orbit as a result of the constant gravitational attraction of fixed magnitude felt by it towards the Earth.

Do you know ?

A parabola is a symmetrical open curve obtained by the intersection of a cone with a plane which is parallel to its side. Mathematically, the parabola is described with the help of a point called the focus and a straight line called the directrix shown in the accompanying figure. The parabola is the locus of all points which are equidistant from the focus and the directrix. The chord of the parabola which is parallel to the directrix and passes through the focus is called latus rectum of the parabola as shown in the accompanying figure.



3.4.1 Period, Radius Vector and Angular Speed:

Consider an object of mass *m*, moving with a uniform speed v, along a circle of radius *r*. Let *T* be the time period of revolution of the object, i.e., the time taken by the object to complete one revolution or to travel a distance of $2\pi r$.

Thus, $T = 2\pi r/v$

$$\therefore \text{Speed } v = \frac{\text{Distance}}{\text{Time}} = \frac{2\pi r}{T} \quad \text{--- (3.48).}$$

During circular motion of a point object, the position vector of the object from centre of the circle is the radius vector $\vec{r}$. Its magnitude is radius *r* and it is directed away from the centre to the particle, i.e., away from the centre of the circle. As the particle performs UCM, this radius vector describes equal angles in equal intervals of time. At this stage we can define a new quantity called angular speed ω which gives the angle described by the radius vector, per unit time. It is analogous to speed which is distance travelled per unit time.

During one complete revolution, the angle described is 2π and the time taken is period *T*. Hence, the angular speed

$$\omega = \frac{\text{Angle}}{\text{time}} = \frac{2\pi}{T} = \frac{(2\pi)}{\left(\frac{2\pi r}{v}\right)} = \frac{v}{r} \quad \text{--- (3.49)}$$

The unit of ω is radian/sec.

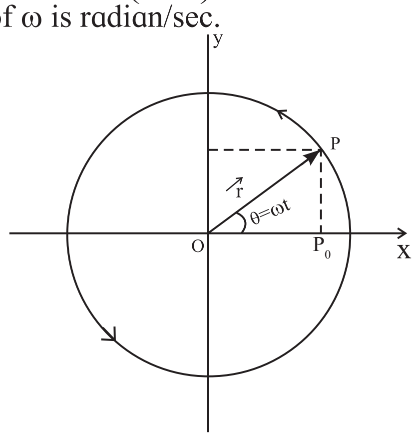


Fig.3.6: Uniform circular motion.

3.4.2 Expression for Centripetal Acceleration:

Figure 3.6 shows a particle P performing a UCM in anticlockwise sense along a circle of radius *r* with angular speed ω and period *T*. Let us choose the coordinates such that this motion is in the *xy*- plane having centre at the origin O. Initially (for simplicity), let the particle be at P_0 on the positive *x*-axis. At a given instant *t*, the radius vector of P makes an angle θ with the *x*-axis.

$$\therefore \theta = \omega t \text{ and so } \frac{d\theta}{dt} = \omega$$

x and *y* components of the radius vector $\vec{r}$ will then be $r\cos\theta$ and $r\sin\theta$ respectively.

$$\therefore \vec{r} = (r\cos\theta)\hat{i} + (r\sin\theta)\hat{j}$$

$$= (r\cos[\omega t])\hat{i} + (r\sin[\omega t])\hat{j} \quad \text{--- (3.50)}$$

Time derivative of position vector $\vec{r}$ gives instantaneous velocity $\vec{v}$ and time derivative of velocity $\vec{v}$ gives instantaneous acceleration $\vec{a}$. Magnitudes of r and ω are constants.

$$\therefore \vec{v} = \frac{d\vec{r}}{dt} = r\left(-\omega\sin[\omega t]\hat{i} + \omega\cos[\omega t]\hat{j}\right)$$

$$= r\omega\left(-\sin[\omega t]\hat{i} + \cos[\omega t]\hat{j}\right) \quad \text{--- (3.51)}$$

$$\therefore \vec{a} = \frac{d\vec{v}}{dt} = r\omega\left(-\omega\cos[\omega t]\hat{i} - \omega\sin[\omega t]\hat{j}\right)$$

$$= -\omega^2\left(r\cos[\omega t]\hat{i} + r\sin[\omega t]\hat{j}\right) = -\omega^2\vec{r} \quad \text{--- (3.52)}$$

Here minus sign shows that the acceleration is opposite to that of $\vec{r}$, i.e., towards the centre. This is the centripetal acceleration.

The magnitude of acceleration,

$$a = \omega^2 r = \frac{v^2}{r} = \omega v \quad \text{--- (3.53)}$$

The force providing this acceleration should also be along the same direction, hence centripetal.

$$\therefore \vec{F} = m\vec{a} = -m\omega^2\vec{r} \quad \text{--- (3.54)}$$

Magnitude of $F = m\omega^2 r = \dfrac{mv^2}{r} = m\omega v$ - (3.55)

Conical pendulum

In a simple pendulum a mass m is suspended by a string of length l and moves along an arc of a vertical circle. If the mass instead revolves in a horizontal circle and the string which makes a constant angle with the vertical describes a cone whose vertex is the fixed point O, then mass-string system is called a conical pendulum as shown in Fig. 3.7. In the absence of friction, the system will continue indefinitely once started.

As shown in the figure, the forces acting on the bob of mass, m, of the conical pendulum are: (i) Gravitational force, mg, acting vertically downwards, (ii) Force due to tension $\vec{T}$ acting along the string directed towards the support. These are the only two forces acting on the bob.

For the bob to undergo horizontal circular motion, (radius r) the resultant force must be centripetal, (directed towards the centre of the circle). In other words vertical gravitational force must be balanced.

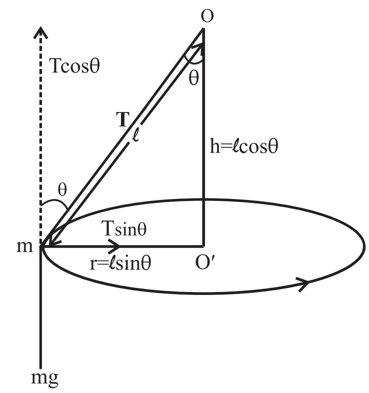


Fig 3.7: Conical pendulum

Thus, we resolve tension $\vec{T}$ into two mutually perpendicular components. Let θ be the angle made by the string with the vertical at any position. The component $T\cos\theta$ is acting vertically upwards. The inclination should be such that $T\cos\theta = mg$, so that there is no net vertical force.

The resultant force on the bob is then $T\sin\theta$ which is radial or centripetal or directed towards centre O' $T\sin\theta = mv^2/r = mr\omega^2$.

$$\tan\theta = \frac{(mv^2/r)}{mg} = \frac{v^2}{rg}$$

Since we know $v = \dfrac{2\pi r}{T}$

$$\therefore \tan\theta = \frac{4\pi^2 r^2}{T^2 rg}$$

$$T = 2\pi\sqrt{\frac{r}{g\tan\theta}}$$

$$T = 2\pi\sqrt{\frac{l\sin\theta}{g\tan\theta}} \quad (\because r = l\sin\theta)$$

$$T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

$$T = 2\pi\sqrt{\frac{h}{g}} \quad (\because h = l\cos\theta) \quad \text{--- (3.56)}$$

where l is length of the pendulum and h is the vertical distance of the horizontal circle from the fixed point O.

Example 3.8: An object of mass 50 g moves uniformly along a circular orbit with an angular speed of 5 rad/s. If the linear speed of the particle is 25 m/s, what is the radius of the circle? Calculate the centripetal force acting on the particle.

Do you know ?

1. The centripetal force is not one of the external forces acting on the object. As can be seen from above, the actual forces acting on the bob are T and mg, the resultant of these is the centripetal force. Conversely, if the resultant force is centripetal, motion must be circular.
2. In planetary motion, the gravitational force between Sun and the planets provides the necessary centripetal force for the circular motion.

Solution: The linear speed and angular speed are related by $v = \omega r$

$\therefore r = v/\omega = 25/5 \text{ m} = 5 \text{ m}$.

Centripetal force acting on the object $= \frac{mv^2}{r} = \frac{0.05 \times 25^2}{5} = 6.25$ N.

Example 3.9: An object is travelling in a horizontal circle with uniform speed. At $t = 0$, the velocity is given by $\vec{u} = 20\hat{i} + 35\hat{j}$ km/s. After one minute the velocity becomes $\vec{v} = -20\hat{i} - 35\hat{j}$. What is the magnitude of the acceleration?

Solution: Magnitude of initial and final velocities =

$= u = \sqrt{(20)^2 + (35)^2}$ m/s

$= \sqrt{1625}$ m/s

$= 40.3$ m/s

As the velocity reverses in 1 min, the time period of revolution is 2 min.

$T = \frac{2\pi r}{u}$, giving $r = \frac{uT}{2\pi}$

$a = \frac{u^2}{r} = \frac{u^2 2\pi}{uT} = \frac{2\pi u}{T} = \frac{2 \times 3.14 \times 40.3}{2 \times 60}$

$= 2.11 \text{ m s}^{-2}$

Internet my friend

1. hyperphysics.phy-astr.gsu.edu/hbase/mot.html#motcon
2. www.college-physics.com/book/mechanics

Exercises

1. Choose the correct option.

i) An object thrown from a moving bus is on example of
   (A) Uniform circular motion
   (B) Rectilinear motion
   (C) Projectile motion
   (D) Motion in one dimension

ii) For a particle having a uniform circular motion, which of the following is constant
   (A) Speed (B) Acceleration
   (C) Velocity (D) Displacement

iii) The bob of a conical pendulum under goes
   (A) Rectilinear motion in horizontal plane
   (B) Uniform motion in a horizontal circle
   (C) Uniform motion in a vertical circle
   (D) Rectilinear motion in vertical circle

iv) For uniform acceleration in rectilinear motion which of the following is not correct?
   (A) Velocity-time graph is linear
   (B) Acceleration is the slope of velocity time graph
   (C) The area under the velocity-time graph equals displacement
   (D) Velocity-time graph is nonlinear

v) If three particles A, B and C are having velocities $\vec{v}_A$, $\vec{v}_B$ and $\vec{v}_C$ which of the following formula gives the relative velocity of A with respect to B
   (A) $\vec{v}_A + \vec{v}_B$ (B) $\vec{v}_A - \vec{v}_C + \vec{v}_B$

(C) $\vec{v}_A - \vec{v}_B$ (D) $\vec{v}_C - \vec{v}_A$

2. Answer the following questions.

i) Separate the following in groups of scalar and vectors: velocity, speed, displacement, work done, force, power, energy, acceleration, electric charge, angular velocity.

ii) Define average velocity and instantaneous velocity. When are they same?

iii) Define free fall.

iv) If the motion of an object is described by $x = f(t)$ write formulae for instantaneous velocity and acceleration.

v) Derive equations of motion for a particle moving in a plane and show that the motion can be resolved in two independent motions in mutually perpendicular directions.

vi) Derive equations of motion graphically for a particle having uniform acceleration, moving along a straight line.

vii) Derive the formula for the range and maximum height achieved by a projectile thrown from the origin with initial velocity $\vec{u}$ at an angel θ to the horizontal.

viii) Show that the path of a projectile is a parabola.

ix) What is a conical pendulum? Show that its time period is given by $2\pi\sqrt{\frac{l\cos\theta}{g}}$, where l is the length of the string, θ is the angle that the string makes with the vertical and g is the acceleration due to gravity.

x) Define angular velocity. Show that the centripetal force on a particle undergoing uniform circular motion is $-m\omega^2\vec{r}$.

3. Solve the following problems.

i) An aeroplane has a run of 500 m to take off from the runway. It starts from rest and moves with constant acceleration to cover the runway in 30 sec. What is the velocity of the aeroplane at the take off ?

[Ans: 120 km/hr]

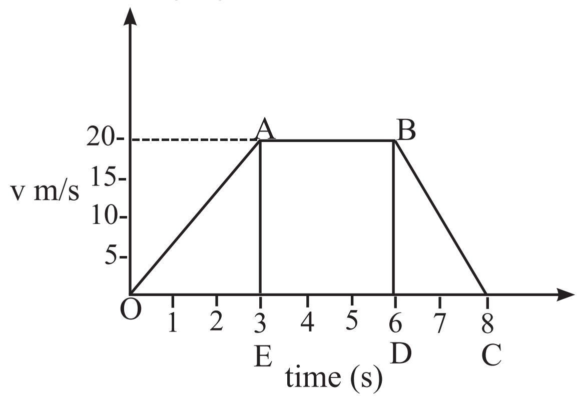
ii). A car moving along a straight road with a speed of 120 km/hr, is brought to rest by applying brakes. The car covers a distance of 100 m before it stops. Calculate (i) the average retardation of the car (ii) time taken by the car to come to rest.

[Ans: 50/9 m/sec^2, 6 sec]

iii) A car travels at a speed of 50 km/hr for 30 minutes, at 30 km/hr for next 15 minutes and then 70 km/hr for next 45 minutes. What is the average speed of the car?

[Ans: 56.66 km/hr]

iv) A velocity-time graph is shown in the adjoining figure.



Determine:

(i) initial speed of the car (ii) maximum speed attained by the car (iii) part of the graph showing zero acceleration (iv) part of the graph showing constant retardation (v) distance travelled by the car in first 6 sec.

[Ans: (i) 0 (ii) 20 m/sec (iii) AB (iv) BC (v) 90 m]

v) A man throws a ball to maximum horizontal distance of 80 meters. Calculate the maximum height reached.

[Ans: 20 m]

vi) A particle is projected with speed v_0 at angle θ to the horizontal on an inclined surface making an angle ϕ ($\phi < \theta$) to the horizontal. Find the range of the projectile along the inclined surface.

[Ans: $R = \frac{2v_0^2\cos\theta\sin(\theta-\phi)}{g\cos^2\phi}$]

vii) A metro train runs from station A to B to C. It takes 4 minutes in travelling from station A to station B. The train halts at station B for 20 s. Then it starts from station B and reaches station C in next 3 minutes. At the start, the train accelerates for 10 sec to reach the constant speed of 72 km/hr. The train moving at the constant speed is brought to rest in 10 sec. at next station. (i) Plot the velocity- time graph for the train travelling from the station A to B to C. (ii) Calculate the distance between the stations A, B and C.

[Ans: AB = 4.6 km, BC =3.4 km]

viii) A train is moving eastward at 10 m/sec. A waiter is walking eastward at 1.2m/sec; and a fly is flying toward the north across the waiter's tray at 2 m/s. What is the velocity of the fly relative to Earth

[Ans: 11.4 m/s, 10° due north of east]

ix) A car moves in a circle at the constant speed of 50 m/s and completes one revolution in 40 s. Determine the magnitude of acceleration of the car.

[Ans: 7.85 m s^{-2}]

x) A particle moves in a circle with constant speed of 15 m/s. The radius of the circle is 2 m. Determine the centripetal acceleration of the particle.

[Ans: 112.5 m s^{-2}]

xi) A projectile is thrown at an angle of 30° to the horizontal. What should be the range of initial velocity (u) so that its range will be between 40m and 50 m? Assume g = 10 m s^{-2}.

[Ans: $21.49 \le u \le 24.03$ m s^{-2}]

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